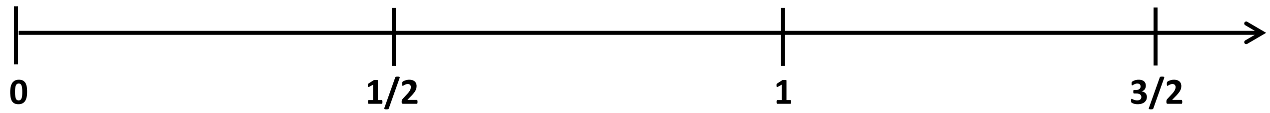
TNCore

Task: Closer to 1 | **4^{th} Grade**

Giana and Tiffany are working with some fractions and the number line below.



A. Giana and Tiffany need to place 3/4 and 4/3 on the number line. Can you place them on the number line and explain why you placed them there?

B. Giana is wondering which fraction is closer to 1. Can you tell Giana which is closer to 1 and explain why?

C. Tiffany notices that 3/4 and 4/3 are special fractions because they are the same numbers, just flipped. She is wondering about other special fractions like 5/6 and 6/5. Is there a way to know which one is closer to 1 for all special fraction pairs? Explain your thinking.

Common Core State Standards for Mathematical Content	Common Core State Standards for Mathematical Practice
3. Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.* 4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).* b. Understand a multiple of *a*/*b* as a multiple of 1/*b*, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Essential Understandings

A situation that can be represented by multiplication has an element that represents the scalar and an element that represents the quantity to which the scalar applies.

A scalar definition of multiplication is useful in representing and solving problems beyond whole number multiplication and division.

The fraction *N*/*D* could be interpreted as *N* copies of the fraction 1/*D* when the whole is divided into D pieces.

Explore Phase

Possible Solution Paths	Assessing and Advancing Questions

<u>Part A:</u> Students place the points on the number line appropriately. ¾ will be to the left of 1, 4/3 should be to the right of 1. (¾ should be visibly closer to 1; if it is not, parts B and C allow the opportunity to revise the placement) For ¾, students may compare to the benchmark fraction ½ and reason that it is halfway between ½ and 1. For 4/3, students may explain this is greater than a whole and so it goes to the right of 1.	<u>Assessing Questions</u> Let's look at your number line. Explain how you knew where to place your points. <u>Advancing Questions</u> How does ¾ compare to a whole? How does 4/3 compare to a whole? Are they more or less? How do you know?
<u>Part B:</u> Students may attempt to make equivalent fractions and compare with a common denominator, 12ths. ¾ = 9/12 while 4/3 = 16/12. 9/12 is 3 parts less than 1 while 16/12 is 4 parts more than 1, so ¾ is closer to 1. Students see ¾ as 3(1/4) = ¼ + ¼ + ¼, ¼ less than a whole. Students see that 4/3 is 4 multiples of 1/3, 4(1/3) = 1/3 + 1/3 + 1/3 + 1/3, 1/3 more than a whole. ¼ < 1/3 so ¾ is closer to 1.	<u>Assessing Questions</u> Tell me what you're thinking to decide this. Do you think one is closer? Why? <u>Advancing Questions</u> What does the denominator tell you? What size are the pieces for the fraction ¾? What size are the pieces for the fraction 4/3? How far away is ¾ from a whole? How do you know? How many ¼'s do you have?
<u>Part C:</u> Students may try a few pairs, like 5/6 and 6/5 or 4/5 and 5/4. Because the smaller fraction has smaller size pieces, and is one piece short of a whole, it will always be closer to 1 when compared to the other fraction that has bigger pieces and is one piece more than a whole. (students should be able to give an informal argument by reasoning about the size of the pieces, or by recognizing the size of the pieces and scaling by the whole number factor, MP3)	<u>Assessing Questions</u> Think about how you answered part B. How did you decide which is closer to 1? How can that help you in part C? <u>Advancing Questions</u> Let's take another example, like 4/5 and 5/4. How do the size of the pieces compare? Will this always be the case for these types of fraction pairs? Why?
Possible Student Misconceptions	
<u>Part A:</u> Students place 4/3 to the right of 3/2 because the 4 and 3 are both greater than the 3 and 2, respectively.	<u>Assessing Questions</u> Tell me why you placed the 4/3 where you did. What things did you consider when deciding where to place it? <u>Advancing Questions</u> What does the denominator tell you about a fraction like 4/3? What does the numerator tell you?

<table>
<tr><td><u>Part B:</u>
Students may think they are the same distance from 1 because they are both one part away from 1.</td><td><u>Assessing Questions</u>
Tell me why you think the points are the same distance from 1. How did you make that determination?
<u>Advancing Questions</u>
Think about ¾. What does the denominator tell you about the number of pieces it takes to make a whole? How many pieces do you have?</td></tr>
<tr><td><u>Part C:</u>
Students may think all special fraction pairs are the same distance from 1 because they are both one part away. Students are not considering the size of the pieces of the fractions.</td><td><u>Assessing Questions</u>
Tell me why you think all fraction pairs like this are the same distance from 1. What did you consider?
<u>Advancing Questions</u>
How do the denominators compare in the special fraction pairs? Will this always happen?
How many pieces are both fractions from 1? How can we tell which is closer to 1?</td></tr>
<tr><td>Entry/Extensions</td><td>Assessing and Advancing Questions</td></tr>
<tr><td>If students can't get started….</td><td><u>Assessing Questions</u>
Which is smaller: ¾ or 4/3? How do you know?
<u>Advancing Questions</u>
How does ¾ compare to 1 – is it greater or less than 1? How do you know?
How does 4/3 compare to 1 – is it greater or less than 1? How do you know?
Which pieces are smaller – fourths or thirds?</td></tr>
<tr><td>If students finish early….</td><td><u>Assessing Questions</u>
How do you know that you have shown that for all fraction pairs, the one less than 1 is closer to 1?
<u>Advancing Questions</u>
Do you think this will be true of other type of fraction pairs, like 5/7 and 7/5? Why or why not?</td></tr>
<tr><td colspan="2">Discuss/Analyze</td></tr>
<tr><td colspan="2">Whole Group Questions</td></tr>
<tr><td colspan="2"><u>Part A:</u>
Who can explain how you decided where to place these two fractions on the number line? Did anyone change his/her placement after working part B? Why? What made you change your mind?</td></tr>
<tr><td colspan="2"><u>Part B:</u>
How did you decide which is closer to 1? What did you consider?</td></tr>
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By comparing only the denominators, what does that tell us? How do the size of the pieces compare? What does the numerator tell us? How many ¼'s do we have? How can this be expressed by repeated addition? Multiplication? How may 1/3's do we have? How can this be expressed by repeated addition? Multiplication? How does this help us decide?
Part C: Do we think that what happened in part B will happen for all special fraction pairs? How could we tell? Who can share his/her thinking? What are some important things to consider in order for us to give an argument?